

SUBORDINATION PRINCIPLE FOR FRACTIONAL EVOLUTION EQUATIONS

Emilia G. Bazhlekova

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Abstract

The abstract Cauchy problem for the fractional evolution equation

$$\mathbf{D}^\alpha u = Au, \quad \alpha > 0, \quad (1)$$

where A is a closed densely defined operator in a Banach space, is investigated. The subordination principle, presented earlier in [J. P r ü s s, *Evolutionary Integral Equations and Applications*. Birkhäuser, Basel - Boston - Berlin (1993)] for general Volterra integral equations, is studied in detail for (1). Besides some additional properties of the subordinated solution operator, we discuss also some new aspects of this principle, such as invertibility, the inverse formula and the Feller semigroup of subordination operators.

Mathematics Subject Classification: 26A33, 47D06, 47D09

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1. Introduction

Consider a linear closed operator A densely defined in a Banach space X . Let $\alpha > 0$ and $n \in \mathbf{N}$ be such that $n - 1 < \alpha \leq n$. Given $x \in X$, we investigate the following Cauchy problem for the *fractional evolution equation* of order α :

$$\mathbf{D}_t^\alpha u(t) = Au(t), \quad u(0) = x, \quad u^{(k)}(0) = 0, \quad k = 1, \dots, n - 1. \quad (FE_\alpha)$$